# BASIC NUMERICAL SKILLS
# MODULE I

The theory of sets was introduced by the German mathematician Georg Cantor in 1870. A set is well defined collection of distinct objects. The term well defined we mean that there exists a rule with the help of which we will be able to say whether a particular object 'belong to' the set or does not belong to the set. The objects in a set are called its members or elements.

**Representation of a Set or Methods of describing a Set**

A set is often representation in two ways:

(1) Roster method or tabular or enumeration method.

(2) Set builder method or Rule or Selector method.

**Tabular Method:** In this method, a set is described by listing the elements, separated by commas and are enclosed within braces. For example the set of first three odd numbers 1,3,5 is represented as : A = {1,3,5}

**Set Builder Method:** In this method, the set is represented by specifying the characteristic property of its elements.

**TYPES OF SETS**

**1. Null Set or Empty Set or Void Set:** A set containing no element is called a null set. It is denoted by { } or ∅ Eg:- the set of natural numbers between 4 and 5.

**2. Singleton or Unit Set:** A Set containing a single element is called singleton set Eg: Set of all positive integers less than 2

**3. Finite Set:** A Set is said to be a finite set if it consist only a finite number of elements. The null set is regarded as a finite set. Eg:- the set of natural numbers less than 10

**4. Infinite Set:** A set is said to be an infinite set if it consists of an infinite number of elements. Eg:- Set of natural numbers.

**5. Equivalent Set:** Two sets A and B are said to be equivalent set if they contain the same number of elements

**6. Equal Set:** Two sets A and B are said to be equal if they contain the same elements.

**7. Sub-Set and Super Set:** If every element of A is an element of B then A is called a subset of B and symbolically we write A⊆B If A is contain in B then B is called super set of A and written as B ⊇A

**8. Power Set :**The collection of all sub sets of a set A is called the power set of A. It is denoted by P(A). In P(A), every element is a set.

**9. Universal Set:** If all the sets under consideration are subsets of a fixed set U, is called universal set. For example A is the set of vowels in the English Alphabet. Then the set of all letters of the English Alphabet may be taken as the universal set.

**10. Disjoint Set:** Two sets A and B are said to be disjoint sets if no element of A is in B and no element of B is in A.

**SET OPERATIONS**

**(1)Union of sets :** The union of two sets A and B is the set of all those elements which belongs to A or to B or to both. We use the notation A∪B to denote the union of A and B.

**(2) Intersection of Sets**

The intersection of two sets is the set consisting of all elements which belong to both A and B. It is denoted by A∩B

**(3) Difference of two sets**

The difference of the two sets A and B is the set of all elements in A which are not in B. It is denoted by A-B or A/B.

**(4) Complement of a set**

Complement of a set is the set of all element belonging to the universal set but not belonging to A. That is complement of union of two sets equal to the intersection of their complements.

**VENN DIAGRAM**

The relationship between sets can be represented by means of diagrams. It is known as Venn diagram. It consists of a rectangle and circles. Rectangle represents the universal set and circle represents any set.

## MATRICES

A matrix is an ordered rectangular array of numbers or functions. It is a rectangular presentation of numbers arranged systematically in rows and columns one number or functions are called the elements of the matrix. The horizontal lines of elements of the matrix are called rows and vertical lines of elements of matrix are called columns.

**Order Of Matrix**

A matrix having 'm' rows 'n' columns are called a matrix of order 'm x n' or simply 'm x n' matrix (read as an 'm' by 'n' matrix)

Types of Matrices

(i) **Rectangular matrix** : Any matrix with 'm' rows and 'n' column is called a rectangular matrix. It is a matrix of Order m x n.

(ii) **Square matrix :** A matrix by which the number of rows are equal to the number of columns, is said to be a square matrix. Thus an m x n matrix is said to be square matrix if m= n and is known as a square matrix of order 'n'.

(iii) **Row matrix** : A matrix having only one row is called a row matrix.

(iv) **Column matrix** : A matrix having only column is called column matrix.

(v) **Diagonal matrix** : A square matrix is said to be diagonal it all elements except leading diagonal are zero. Elements $a_{11}$, $a_{22}$, $a_{33}$ etc. termed as leading diagonal of a matrix.

(vi) **Scalar Matrix** : A diagonal matrix is said to be scalar matrix, if its diagonal elements are equal.

(vii) **Unit matrix of identity matrix** : A diagonal matrix in which diagonal elements are 1 and rest are zero is called Unit Matrix or identity matrix. It is denoted by 1.

(viii) **Null Matrix or Zero matrix:** A matrix is said to be zero or null matrix if all its elements are zero

(ix) **Triangular matrix:** If every element above or below the leading diagonal is zero, the matrix is called Triangular matrix. It may be upper triangular or lower triangular. In upper triangular all elements below the leading diagonal are zero and in the lower triangular all elements above the leading diagonal are zero.

(x) **Symmetric matrix :** Any square matrix is said to be symmetric if it is equal to transpose. That is, $A = A^t$ Transpose of a matrix as a matrix obtained by interchanging its rows and columns. It is denoted by $A^t$ or $A^1$.

(xi) **Skew Symmetric Matrix** : Any square matrix is said to be skew symmetric if it is equal to its negative transpose. That is $A = -A^t$

**<u>Operation of matrices</u>**

Operation of matrices relate to the addition of matrices, difference, multiplication of matrix by a scalar and multiplication of matrices.

**Addition of matrices** : If A and B are any two matrices of the same order, their sum is obtained by the elements of A with the corresponding elements of B.

**Difference of Matrices** : if A and B are, two matrices of the same order, then the difference is obtained by deducting the element of B from A.

**Multiplication of a Matrix by a Scalar**: The elements of Matrix A is multiplied by any value (ie. K) and matrix obtained is denoted by K

**Multiplication of two matrices**

For multiplication, take each row of the left hand side matrix with all colums of the right hand side matrix.

**Determinants**

A determinant is a compact form showing a set of numbers arranged in rows and columns, the number of rows and the number of columns being equal. The number in a determinant are known as the elements of the determinant.

Matrices which are not square do not have determinants. Determinant of Square matrix of order 1 The determinants of 1 x 1 matrix A [a] is denoted by |A| or det. A (i.e. determinant of A) and its value is a. Determinant of Square matrix of order 2

Singular and Non-singular matrices – A square matrix 'A' is said to be singular if its determinant value is zero. If $|A| \neq 0$, then A is called non-singular.

Minor elements of a matrix: Minor element is the determinant obtained by deleting its rows and the column in which element lies.

**Cofactor of an element**

Co-factor of an element is obtained by multiplying the minor of that element with $(-1)^{(i+j)}$ . where i = the row in which the element belongs, s = the column in which the element belongs. Co-factor of an element = Minor of an element X $(-1)^{i+j}$

**Ad joint Matrix**

Ad joint of a given matrix is the transpose of the matrix formed by co-factors of the elements. It is denoted by Adj A.

**Invertible Matrix and Inverse of a Matrix**

Let A be a square matrix of order n, if there exist a square matrix B of order n, such that AB = BA = I Then A is said to be convertible and B is called on inverse of A and A is called inverse of B Where I = Identity Matrix Inverse of A is denoted by $A^{-1}$

## MODULE II

## THEORY OF EQUATIONS

An equation is a statement of equality between two expressions. For eg:- $x + 2 = 5$. An equation contains one or more unknowns.

Types of Equations

**1)Linear Equation:** It is an equation when one variable is unknown. For example $2x + 3 = 7$

**2) Simultaneous equations in two unknowns:** For solving the equations, firstly arrange the equations. For eliminating one unknown variable, multiply the equation 1 or 2 or both of them with certain amount and then deduct or add some equation with another, we get the value of one variable. Then substitute the value in the equation, we get the values of corresponding variable.

**3) Simultaneous Equations in three unknowns:** Firstly, eliminate one of the unknown from first two equations. Then eliminate the same unknown from second and third equations. Then we get two equations. Solve such equations, we get the values of *x, y* and *z*.

**4) Quadratic equations:** The equation of the form $ax^2 + bx + c = 0$ in which a, b, c are constant is called a quadratic

equation in x. Here x is the unknown. Solution of quadratic equations There are three methods to solve a quadratic equation.

(1) Method by formula
(2) Method of factorization
(3) Method of completing the square

**Quadratic formula method**

One general quadratic equation is $ax^2 + bx + c = 0$ Then $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

# MODULE – III

## PROGRESSIONS

### Arithmetic Progression

A series is said to be in Arithmetic Progression, it its terms continuously increase or decrease by a constant number . It is a series, in which each term is obtained by adding or deducting a constant number to the preceding term. The constant number is called common difference of the progression and is denoted by 'd'. It is the difference between the two term of the series i.e., the difference between second term and first term or third term and second term and so on. The first term of an A.P. is usually denoted by 'a'. One general form of an A.P is *a*, *a+ d, a+2d, a+ 3d*, …….

For example

(i) The sequence 1, 3, 5, 7, . . . . . is an A.P whose first term is 1 and d = 2

(ii) The sequence -5, -2, 1, 4, 7, . . . . , whose 'a' = -5, d = 3

### General term of an AP or $n^{th}$ term

Let '*a*' be the first term and '*d*' be the common difference of an A.P, then $a_n$ denotes the $n^{th}$ term of the A.P.

$$a_n = a + (n1)\, d$$

n = number of term in a series.

### Sum of n terms of an A.P

Let $S_n$ denotes the sum of '*n*' terms of an A.P, whose first term is '*a*' and common difference is '*d*'. $S_n = n/2\ [2a + (n - 1)\, d]$

$2a = a + a$ or $2 \times a$

### Arithmetic Mean (A.M)

Given two numbers a and b, we can insert a number A between them, so that a, A, b is an A.P. Such a number A is called the Arithmetic Mean of the number a and b. We can insert as many numbers as we like between them. Let A, $A_2$, $A_3$ . . . . . $A_n$ be 'n' numbers between a and b,

Then

$A_1 = a + d$

$A_2 = a + 2d$

$A_3 = a + 3\, d$

$A_n = a + nd$

### Geometric Progression

A series is said to be in G.P if every term of it is obtained by multiplying the previous term by a constant number. This constant number is called common ratio, denoted by 'r'. r = second term / first term or third term by second term etc.

The first term of a G.P is usually denoted by a. The general form of a G.P is usually denoted by a. The general form of a G.P is a, ar, $ar_2$, $ar_3$ ….. If the number of terms of a G.P is finite, it is called a finite G.P, otherwise it is called an infinite G.P.

### General term of a G.P or $n^{th}$ term of a G.P

Let 'a' be the first term and 'r' be the common ratio of a G.P, then $a_n = ar_{n1}$

### Geometric Mean

One geometric mean of two positive numbers *a* and b is the number $\sqrt{ab}$. Therefore, the geometric mean of 2 and 8 is 4. We can insert as many numbers as we like between *a* and b to make the sequence in a G.P. Let $G_1$, $G_2$, $G_3$, ……. $G_n$ be 'n' number between a and b, then $G_1 = ar$, $G_2 = ar_2$, $G_3 = ar_3$, $G_n = ar_n$

## MATHEMATICS OF FINANCE

**Simple interest:** It is the interest calculated on principal amount at the fixed rate . Simple Interest = $\frac{Pnr}{100}$

Where P = Principal amount, n = number of year, r = rate of interest per annum.

Amount at the end of **nth** year = $\mathbf{P + \frac{Pnr}{100}}$ or $\mathbf{P(1 + \frac{Pnr}{100})}$ or principal amount + interest

**Compound Interest:** Compound interest means interest calculated on principal amount plus interest. Let 'p' be the principal 'r' be the rate of interest (compound) p.a., 'n' be the number of years then,

Amount = $P\ (1 + \frac{r}{100})^n$

Total interest = A – P

### COMPOUNDING HALF YEARLY OR QUARTERLY

- When interest is compounded half yearly, then r = $\frac{r}{2}$, n = 2n.
- When interest is compounded quarterly, then r = $\frac{r}{4}$ , n = 4n.
- When interest is compounded monthly, then r = $\frac{r}{12}$ , n = 12n.

# MODULE IV

## MEANING AND DEFINITIONS OF STATISTICS

The word statistics is derived from the Latin word 'Status' or Italian word 'Statista' or German word 'Statistik' which means a Political State. It is termed as political state, since in early years, statics indicates a collection of facts about the people in the state for administration or political purpose.

Statistics has been defined either as a singular non or as a plural noun. Definition of Statistics as Plural noun or as numerical facts:- According to Horace Secrist, 'Statistics are aggregates of facts affected to a marked extent by multiplicity of causes numerically expressed, enumerated or estimated according to a reasonable standard of accuracy, collected in a systematic manner for a predetermined purpose and placed in relation to each other'.

Definition of Statistics as a singular noun or as a method:- According to Seliman, " Statistics is the science which deals with the methods of collecting classifying, comparing and interpreting numerical data collected, to know some light on any sphere of enquiry".

### Characteristics of Statistics

(1) Statistics show be aggregates of facts
(2) They should be affected to a marked extent by multiplicity of causes.
(3) They must be numerically expressed.
(4) They should be enumerated or estimated according to a reasonable standard of accuracy.
(5) They should be collected in a systematic manner.
(6) They should be collected for a predetermined purpose.
(7) They should be placed in relation to each other.

### Function of Statistics

The following are the important functions of statistics:

1. **It simplifies complexity**:- Statistical methods make facts and figures easily understandable form. For this purpose Graphs and Diagrams, classification, averages etc are used.
2. **It presents facts in a proper form**:- Statistics presents facts in a precise and definite form.
3. **It facilitates for comparison**:- When date are presented in a simplified form, it is easy to compare date.

4. **It facilitates for formulating policies:-** Statistics helps for formulating policies for the companies, individuals, Govt. etc. it is possible only with the help of date presented in a suitable form.

5**. It tests hypothesis:-** Hypothesis is an important concept in research studies. Statistics provides various methods for testing the hypothesis. The important tests are Chi – square, Z-test, T-test and F-test.

6. **It helps prediction or forecasting:-** Statistical methods provide helpful means of forecasting future events.

7. **It enlarges individual's knowledge**:- When data are presented in a form of comparison, the individuals try to find out the reasons for the variations of two or more figures. It thereby helps to enlarge the individual's knowledge.

8. It measures the trend behavior:- Statistics helps for predicting the future with the help of present and past data. Hence plans, programs, and policies are formulated in advance with the help of statistical techniques.

**Scope of Statistics or importance or utility of statistics.**

(1) **Statistics in Business:-** Statistics is most commonly used in business. It helps to take decision making of the business. The statistical data regarding the demand and supply of product can be collected and analyzed to take decisions. The company can also calculate the cost of production and then the selling price. The existing firms can also make a comparative study about their performance with the performance of others through statistical analysis.

(2) **Statistics in Management:-** Most of the managerial decisions are taken with the help of statistics. The important managerial activities like planning, directing and controlling are properly executed with the help of statistical data and statistical analysis. Statistical techniques can also be used for the payment of wages to the employees of the organization.

(3) **Statistics in economics**:- Statistical data and methods of statistical analysis render valuable assistance in the proper understanding of the economic problems and the formulation of economic policy.

(4) **Statistics in banking and finance:-** Banking and financial activities use statistics most commonly.

(5) **Statistics in Administration:-** The govt. frames polices on the basis of statistical information.

(6) **Statistics in research:-** Research work are undertaken with the help of statistics.

**Limitation of statistics**

(1) Statistics studies only numerical data

(2) Statistics does not study individual cases.

(3) Statistical result are true only an average.

(4) Statistics does not reveal the entire story of the problem.

(5) Statistics in only one of the methods of study a problem.

(6) Statistics can be misused.

**Statistical Enquires or Investigation**

Statistical Investigation is concerned with investigation of some problem with the help of statistical methods. It implies search for knowledge about some problems through statistical device.

**Different stages in statistical enquiry are:**

(1) Planning the enquiry

(2) Collection of data.

(3) Organization of data.

(4) Presentation of data.
(5) Analysis of data.
(6) Interpretation of data.

(1) **Planning the enquiry:-** The first step in statistical investigation is planning. The investigator should determine the objective and scope of the investigation. He should decide in advance about the type of enquiry to be conducted, source of information and the unit of measurement.

**Object and scope**:- The objective of the Statistical enquiry must be clearly defined. Once the objective of enquiry has been determined, the next step is to decide the scope of enquiry. It refers to the coverage of the enquiry.

**Source of information:-** After the purpose and scope have been defined, the next step is to decide about the sources of data. The sources of information may be either primary or secondary.

**Types of enquiry**:- Selection of type of enquiry depends on a number of factors like object and scope of enquiries, availability of time, money and facilities. Enquiries may be (1) census or sample (2) original or repetitive (3) direct or indirect (4) open or confidential (5) General or special purpose.

**Statistical unit:-** The unit of measurements which are applied in the collected data is called statistical unit. For example ton, gram, meter, hour etc.

**Degree of accuracy:-** The investigator has to decide about the degree of accuracy that he wants to attain. Degree of accuracy desired primarily depends up on the object of an enquiry.

**Cost of plan:-** An estimate of the cost of the enquiry must be prepaid before the commencement of enquiry.

(2) **Collection of data**:- Collection of data implies accounting and systematic recoding of the information gathered in a statistical investigation. Depending on the source, the collected statistical data are classified under two categories namely primary data and secondary data.

(3) **Organization of data**:- Organization of data implies the arrangement and presentation of data in such a way that it becomes easy and convenient to use them. Classification and tabulation are the two stages of organizing data.

(4) **Presentation of data:-** They are numerous ways in which statistical data may be displayed. Graphs and diagrams are used for presenting the statistical data.

(5) **Analysis data:-** Analysis of data means critical examination of the data for studying characteristics of the object under study and for determining the pattern of relationship among the variables.

(6) **Interpretation of data:-** Interpretation refers to the technique of drawing inference from the collected facts and explaining the significance.

**<u>Classification according to variables</u>**

Data are classified on the basis of quantitative characteristics such as age, height, weight etc.

**Geographical Classification**:- Classified according to geographical differences.

**Chronological Classification:-** Classified according to period wise.

**<u>Frequency Distribution</u>**

A frequency distribution is an orderly arrangement of data classified according to the magnitude of observations. When data are grouped into classes of appropriate size indicating the number of observations in each class we get a frequency distribution.

**Components of frequency Distribution**

(1) Class and class interval
(2) Class limits

**Methods of classification**

(1) Classification according to attributes.
(2) Classification according to variables.

**Classification according to attributes**: Under this methods the data are classified on the basis of attributes. For example literacy, unemployment etc. are attributes.

Following are the classification under this method.

1. Simple classification
2. Manifold classification

In simple classification the data are divided on the basis of only one attributes. In manifold classification the data are classified on the basis more than one attributes. For example population is divided on the basis of sex and literacy.

3. Class mark
4. Class boundaries
5. Magnitude of class interval
6. Class frequency.

**Tabulation**

Tabulation is an orderly arrangement of data in rows and columns. It is a moment of presentation of data.

**Objectives**

1. To simplify complex data
2. To facilitate comparison
3. To facilitate statistical analysis
4. To save time
5. To economies space

**Part of a table**

1. Table number
2. Title of the table
3. Caption -------- i.e. column headings
4. Sub ---------- i.e. row heading
5. Body
6. Head note
7. Foot note
8. Source data.

**Collection of data**

On the basis of source, data can be collected from primary and secondary source. Primary data Primary data are those collected by the investigator himself. May are original in character. May are truthful and suit for the purpose. But the collection is very expensive and time consuming.

**Methods of collection of primary data**

1. **Direct personal interview:-** In this method investigator collection the data personally. He was to meet the people for collecting the data. This method is suitable:

a) When the area of investigation is limited
b) When higher degree of accuracy is leaded.
c) When the results of investigation to be kept confidential.

2. **Indirect oral investigation:-** Under this method, information are collected from third parties who are is touch with the facts under enquiry.

3. **Schedules and Questionnaires methods:-** Under this method, a list of questions called questionnaire is prepared and information are called from various sources. It is a printed list of questions to be filled by the information. But schedule is filled by the enumerator.

**Essentials of a good questionnaire**

(1) The person conducting the survey much introduce himself.
(2) The number of questions should be kept to the minimum.
(3) The question should be as short as possible and simple.
(4) The questions must be arranged in logical order.
(5) The questions should be clear.

(6) Personal questions should be avoided.
(7) Questions should be in the nature of yes or no type.
(8) Questions must be of convenient size and easy to handle.
(9) Questions should be attractive.
(10) Instructions should be given for filing up the form.

**Secondary data**

Secondary data are those data which are collected by someone for this purpose. Secondary data are usually in the shape of finished product. The collection of secondary data is less expensive and less time consuming. Secondary data are collected from published and unpublished sources.

**Difference between Primary and Secondary data**

1. Primary data are original character. But secondary data are not original, they are collected by somebody else.
2. Primary data are in the shape of raw material. But secondary data are in the shape of finished product.
3. Collection of primary data is expanse and time consuming. But collection of secondary data is less expensive and less time consuming.
4. Primary data will be usually adequate and suitable. But secondary data need not be adequate and suitable for the purpose.

**Sampling:** Sampling is the process obtaining information about an entire population by examining only a part of it. It is the examination of the regenerative items and conclusion of draw for all items coming in that group.

**Methods of sampling or techniques of sampling**

1. Probability sampling or random sampling
2. Non probability sampling

**Probability sampling**

Under this method, each items has an equal chance for being selected. Following are the random sampling.

**(1) Simple random sampling:** A simple random sample is a sample selected from a population in such a way that every item of the population has an equal chance of being selected. The selection depends on chance. Eg. Lottery methods.

**(2) Systematic sampling:** This method is popularly used in those cases where complete list of the population from which sample is to be drawn is available. Under this method the items in the population are included in intervals of magnitude K. From every interval select an item by simple random sample method.

**(3) Cluster sampling:** Cluster sampling consists in forming suitable clusters of units. All the units is the sample of clusters selected are surveyed.

**(4) Quota sampling:** In this method each investigator engaged in the collection of data is assigned a quota for investigation.

**(5) Multi stage sampling:** This is a sampling procedure carried but in several stages. In multistage sampling, firstly units selected by suitable methods of sampling. From among the selected units, sample is drawn by some suitable methods. Further stages are added to arrive at a sample of the desired units

**Non probability sampling**

**1. Judgment sampling:-** Under this sampling investigator exercise this discretion in the matter of selecting the items that are to be included in the sample.

**2. Convenience Sampling**:- Convenience sampling is one in which a sample is obtained by selecting such units of the universe which may be conveniently located.

**Organization of data**

Organizing data mean, the arrangement and presentation of data. Classification and tabulation are the two stages of organizing data.

**Classification**

The process of arranging data in groups or classes according to similarities called classification.

**Objects of classification**

1. To simplify the complexity of data.
2. To bring out the points of similarity of the various items.
3. To facilitate comparison.
4. To bring out relationship.
5. To provide basis for tabulation.

**Graphs and Diagrams**

Graphs and diagrams is one of the statistical methods which simplifies the complexity of quantitative data and make them easily understandable.

**Importance of Diagrams & Graphs**

1. Attract common people
2. Presenting quantitative facts in simple.
3. They have a great memorizing effect.
4. They facilitate comparison of data.
5. Save time in understanding data.
6. Facts can be a understood without mathematical calculations.

**Limitations**

1. They can present only approximate values.
2. They can represent only limited amount of information.
3. They can be misused very easily.
4. They are not capable of further mathematical treatment.
5. They are generally useful for comparison purpose only.

**General rules for constructing Diagrams**

1. Title
2. Proportion between width and height.
3. Selection of scale
4. Foot note
5. Index
6. Neatness and cleanliness
7. Simplicity
8. Attractiveness

**Types of Diagrams**

1. Dimensional Diagrams
2. Cartograms
3. Pictograms

**Dimensional Diagrams:** Dimensional Diagrams are those diagrams which show information in terms of length, height, area or volume. They are one dimensional two dimensional or three dimensional.

**One Dimensional Diagram:** In one dimensional diagram the height will represent the magnitude of observations. Must commonly use one dimensional diagrams are line diagram and Bar diagram.

**Line Diagram:** Line diagrams are one dimensional diagrams. They are drawn to represent values of a variable.

**Bar Diagrams:** In a bar diagram only the length is considered. The width of the bar is not given any importance.

Following are the important types of bar diagrams.

(1) **Simple bar diagram**: Simple bar diagram represents only one variable. For example height, weight, etc.

**2) Multiple Bar Diagram:** Two or more interrelated data are represented in a multiple bar diagram. In order to identity the data, the bars should be differentiated with colors or shades.

**3) Sub Divided Bar Diagram:** In the sub

divided bar diagram each bar is subdivided into two or more parts. Each part may explain different characters.

**4) Percentage Bar Diagrams:** In percentage bar diagram the length of all the base are equal ie each bar represent 100 percent. The component parts are expressed as percentage to the whole.

**Two Dimensional Diagram:** In two dimensional diagram the length as well as width have to be considered. The most commonly used two dimensional diagrams is pie diagram, Rectangles, Squares, Circles etc are also two dimensional diagrams.

**Pie Diagrams:** Pie diagrams are used when the aggregate and their divisions are to be shown together. The aggregate is shown by means of a circle and divisions by the sectors of the circle. Eg:, the selling price of a product can be divided into various segments like factory cost, admin cost, selling cost and profit. These segments are converted into percentage in order to represent in the pie diagram. In order to prepare the pie diagram, each percentage outlay must be multiplied by 3.6, since the pie diagram contain 360 scale.

**Three Dimensional Diagrams:** Three dimensional diagrams are prepared in the form of cubes, spheres, cylinders etc. In these diagrams width, length and breadth are important.

**Cartograms:** Cartograms means the presentation of data in a geographical basis. It is otherwise called as statistical maps. The quantities on the map may be shown through shades, dots or colours etc.

**Pictograms:** Under the pictograms, data are represented in the form of an appropriate pictures most suited for the data.

## GRAPHS

### Types of Graphs

(1) Graphs of Frequency Distribution
(2) Graphs of Time Series

### Graphs of Frequency Distribution

A frequency distribution can be presented graphically in any of the following ways:

(1) Histogram
(2) Frequency Polygon
(3) Frequency Curves
(4) Ogive or cumulative frequency curves.

**Histogram:** A histogram is a graph of frequency distributions. A histogram consists of bars erected upon the class interval columns. While constructing histogram, the variable is always taken on the x-axis and the frequency on the y-axis. The width of the bars in the histogram will be proportional to the class interval.

**Frequency Polygon:** It is a curve instead of bars. There are two methods for constructing frequency polygon. First, histogram should be drawn and mark mid-point of upper side of each bar and join such joints by a curve. In the second method, first of all plot the frequencies corresponding to midpoints of various class intervals. Then join all the plotted points to get the frequency polygon curve.

**3) Ogive or Cumulative Frequency Curve:** A frequency distribution when cumulated, we get cumulative frequency distribution and curve drawn is known as ogive. An ogive can either less than ogive or more than ogive. Less than ogive curve is drawn on the basis of less than cumulative frequency distribution and more than ogive is drawn on the basis of more than cumulative frequency distribution.

**Measures of central tendency or Averages**

An average is a single value that represents a group of values. It represents the whole series and conveys general idea of the whole group.

**Characteristics of a good average**

(1) Clearly defined
(2) Easy to understand
(3) Simple to compute
(4) Based on all items
(5) Not be unduly affected by extreme observations.
(6) Capable of further algebraic treatment
(7) Sampling stability.

**Types of averages**

1) Arithmetic Mean
2) Median
3) Mode
4) Geometric mean
5) Harmonic Mean

**Arithmetic Mean (AM)**

It is the value obtained by adding together all the items and by dividing the total number of items. Arithmetic mean may either be (1) Simple arithmetic Mean or (2) Weighted arithmetic Mean

**Simple Arithmetic Mean:** It is the mean of items which give equal importance to all items. It is denoted by $\overline{x}$

$$\overline{x} = \sum \frac{x}{n}$$

Where = Sum of given variables N = Number of items

**Calculation of Arithmetic Mean**

**a) Individual Series :**

( i) **Direct Method**

$$\overline{x} = \sum \frac{x}{n}$$

(ii) **Short Cut Method**

$$\overline{x} = A + \frac{\sum d}{N}$$

A = Assumed mean
D = X – A
n = total number of items

**(b)Discrete Series**

(i) **Direct Method**

$$\overline{x} = \frac{\sum fx}{N}$$

(ii) **Short Cut method**

$$\overline{x} = A + \frac{\sum fd}{N}$$

d = X – A

(iii) **Step deviation method**

$$\overline{x} = A + \frac{\sum fd'}{N} X C$$

c = common factor
d' = x-a/c

**(c) Continuous Series**

(i) **Direct method**

$$\overline{x} = \frac{\sum fm}{N}$$

m = midpoint of X
N = Total frequency

(ii) **Short cut method**

$$\overline{x} = A + \frac{\sum fd}{N}$$

d = m – A

(iii) **Step deviation method:**

$$\overline{x} = A + \frac{\sum fd'}{N} X C$$

C = Common factor or class interval
d ' = m- A/ c

## MODULE V

**Weighted Mean**

Weighted means are obtained by taking in to account of weights. Each value is multiplied by its weight and total is divided by the total weight to get weighted mean.

$$\overline{x}_w = \frac{\sum wx}{n}$$

$\overline{x}_w$ = weighted A.M.

w = weight, x = given variable

**Median**

Median is the middle value of the series. When the series are arranged in the ascending order or descending order Median is a positional average.

**Calculation of Median**

**Individual series**

Firstly arrange the series. Median = Size of $(\frac{n+1}{2})\ th$ item

**Discrete series**

Median = Size of $(\frac{n+1}{2})\ th$ item.

**Continuous series**

Median Class = $\frac{N}{2}$

$$\text{Median} = L1 + \frac{\frac{N}{2} - c.f}{f}\ X\ C$$

L1= Lower limit of median class

c.f = cumulative frequency of preceding median class

f = frequency of median class

C = Class interval

**Mode**

Mode is the value of item of series which occurs most frequently.

**Mode in individual series**

In the case of individual series, the value which occurs more number of times is mode. When no items appear more number of times than others, then mode is the ill-defined. In this case : Mode = 3 median – 2 mean

**Mode in discrete series:** In the case of discrete series, the value having highest frequency is taken as mode.

**Mode in continuous series:** Mode lies in the class having the highest frequency.

$$\text{Mode} = l1 + \frac{(f1 - f0)\text{x c}}{2f1 - f0 - f2}$$

L1 = lower limit of the model class

F1= frequency of the model class

F0, f2= frequency of class preceding and succeeding modal class.

**Geometric Mean**

Geometric mean is defined as the square root of the product of those in values. G.M = *Antilog* $\left(\frac{\sum logx}{n}\right)$

**G.M in Individual series**

G.M = *Antilog* $\left(\frac{\sum logx}{n}\right)$

**G.M in Discrete series**

G.M = *Antilog* $\left(\frac{\sum flogx}{n}\right)$

**G.M in continuous series**

G.M = *Antilog* $\left(\frac{\sum flogx}{n}\right)$

x = midpoint of x

**Harmonic Mean**

Harmonic mean is defined as the reciprocal of the mean of the reciprocals of those values. It applied in averaging rates, times etc.

H.M = $\frac{n}{\sum 1/x}$

**H.M in Discrete series**

H.M = $\frac{n}{\sum f(1/x)}$

**H.M in continuous series**

H.M = $\frac{n}{\sum f(1/x)}$

*x* = midpoint of x

**MEASURES OF DISPERSION OR VARIABILITY**

Dispersion means a measure of the degree of deviation of data from the central value. Measures of Dispersion are classified into (1) Absolute Measures (2) Relative Measures. Absolute Measures of dispersion are expressed in the same units in which data are collected. They measure variability of series. Various absolute measures are:

(i) Range

(ii) Quartile Deviation

(iii) Mean Deviation

(iv) Standard Deviation

Relative measure is also called coefficient of dispersion. They are useful for comparing two series for their

variability. Various relative measures are:
(i) Coefficient Range
(ii) Coefficient of Quartile Deviation
(iii) Coefficient of Mean Deviation
(iv) Coefficient of Variation

**RANGE:** The range of any series is the difference between the highest and the lowest values in the series.

Range = H – L
H = Highest variable
L = Lowest variable

Coefficient of Range = $\frac{\mathbf{H-L}}{\boldsymbol{H+L}}$

**QUARTILE DEVIATION:** Quartile Deviation is defined as the half distance between the third and first quartiles.

Quartile Deviation = $\frac{\boldsymbol{Q3-Q1}}{\mathbf{2}}$

Coefficient of Quartile Deviation = $\frac{\boldsymbol{Q3-Q1}}{\boldsymbol{Q3+Q1}}$

**Quartile Deviation in Individual Series**

Quartile Deviation = $\frac{\boldsymbol{Q3-Q1}}{\mathbf{2}}$

$Q_1$ = size of $\frac{\boldsymbol{n+1}}{\mathbf{4}}$ th Item

$Q_3$ = size of $\mathbf{3}\left(\frac{\boldsymbol{n+1}}{\mathbf{4}}\right)$th item

**Quartile Deviation in Discrete Series**

Quartile Deviation = $\frac{\boldsymbol{Q3-Q1}}{\mathbf{2}}$

$Q_1$ = size of $\frac{\boldsymbol{N+1}}{\mathbf{4}}$ th Item

$Q_3$ = size of 3 $\left(\frac{\boldsymbol{N+1}}{\mathbf{4}}\right)$ th item

**In Continuous series**

Quartile Deviation = $\frac{\boldsymbol{Q3-Q1}}{\mathbf{2}}$

$Q_1 = L1 + \frac{\frac{N}{4} - c.f}{f} \; X \; C$

$Q_3 = L1 + \frac{3 \; X \; \frac{N}{2} - c.f}{f} \; X \; C$

**MEAN DEVIATION**

Mean Deviation is defined as the arithmetic mean of deviations of all the values in a series from their average. The average may be mean, median or mode. Mean Deviation = $\frac{\boldsymbol{\Sigma|d|}}{\boldsymbol{n}}$

Where |d| = deviation from an average without sign

**Mean Deviation in Individual Series**

Mean Deviation = $\frac{\boldsymbol{\Sigma|d|}}{\boldsymbol{n}}$

Coefficient of Mean Deviation = $\frac{M.D}{Average}$

Average = Mean, Median or Mode from which the deviation is taken

**Mean Deviation in Discrete Series**

Mean Deviation = $\frac{\boldsymbol{\Sigma f|d|}}{\boldsymbol{n}}$

Coefficient of Mean Deviation = $\frac{M.D}{Average}$

**Mean Deviation in Continuous Series**

Mean Deviation = $\frac{\boldsymbol{\Sigma f|d|}}{\boldsymbol{n}}$

**STANDARD DEVIATION**

Standard Deviation is defined as the square root of the mean of the squares of the deviations of individual items from their arithmetic mean. It is denoted by σ (sigma) = $\sqrt{\frac{\boldsymbol{\Sigma}(x-\overline{x})2.}{n}}$

**Standard Deviation in Individual Series**

$\sigma = \sqrt{\frac{\Sigma x^2.}{n} - (\frac{\Sigma x}{n})2}$ or $\sqrt{\frac{\Sigma(x-\overline{x})2.}{n}}$

Coefficient of variation = $\frac{\sigma}{x}$ x 100

**Standard Deviation in Discrete Series**

$\sigma = \sqrt{\frac{\Sigma f x^2.}{N} - (\frac{\Sigma fx}{N})2}$

**Shortcut method:**

$\sqrt{\frac{\Sigma f d^2.}{N} - (\frac{\Sigma fd}{N})2}$

d = x - A

**Standard Deviation in Continuous Series**

(i) **Direct Method:**

$\sigma = \sqrt{\frac{\Sigma f x^2.}{N} - (\frac{\Sigma fx}{N})2}$

x = mid point of X

(ii) **Shortcut method:**

$\sqrt{\frac{\Sigma f d^2.}{N} - (\frac{\Sigma fd}{N})2}$

d = m – A or x – A

(iii) **Step Deviation method:**

$\sqrt{\frac{\Sigma f d'^2.}{N} - (\frac{\Sigma fd'}{N})2}$

**d' =** $\frac{\mathbf{d}}{c}$, c = class interval.

**VARIANCE**

Variance is defined as the mean of the squares of the deviations of all the values in the series from their mean. It is the square root of the Standard Deviation.

Variance = $\sigma^2$

**Merits of S.D**

1. S.D. is based on all the values of a series.
2. It is rigidly defined
3. It is capable of further mathematical treatment.
4. It is not much affected by sampling fluctuations.

**Demerits**

1. It is difficult to calculate.
2. Signs of the deviations are not ignored.

**Measures of skewness**

Skewness means lack of symmetry when a frequency distribution is not symmetrical, it is said t be asymmetrical or skewed. In the case of a skewed distribution, the mean, median and mode are not equal. Similarly for a skewed distribution $Q_1$ and $Q_3$ will not be equidistant from median. It is an asymmetrical distribution. It has a long tail on one side and a start tail on the other side. A distribution is said to be skewed when:

(1) Mean, media and mode are not equal.
(2) $Q_1$ and $Q_3$ are not equidistant from median.
(3) Frequencies on either side of mode are not equal.
(4) The frequency curve has longer tail on the left side or on the right side.

**Positive and Negative Skewness**

Skewness is said to be positive when the mean is greater than the median and median is greater than mode. More than half area falls to right side of the highest ordinate.

Skewness is said to be negative when the mean is less than median and the median is less than mode. In this case curve is skewed to the left more than half the area falls to the left of the highest ordinate.

Symmetric Distribution
Positively skewed
Negatively skewed

**Measures of skewness**

**Karl Pearson's skewness**

$$\text{Skewness} = \frac{Mean - median}{\sigma}$$

**Bowley's measure of skewness**

$$\text{Skewness} = \frac{Q3 + Q1 - 2median}{Q3 - Q1}$$

**Kelley's measure of Skewness**

$$\text{skewness} = \frac{P90 + P10 - 2median}{P90 - P10}$$

**Measure of skewness Based on Moments**

$$\text{Skewness} = \frac{M^3}{\sqrt{M2^3}}$$

**Kurtosis**

Kurtosis is a measure of peaked ness. It refers a distribution which is relatively fetker than the normal curve. When a frequency curve is more peaked than the normal curve, it is called lepto kurtic and when it is more flat topped than the normal curve it is called platy kurtic. When a curve is neither peaked nor plat topped, it is called meso kurtic normal.

Lepto Kurtic

Meso Kurtic

Platy Kurtic

**Lorenz Curve**

Lorenz curve is a graphical method of studying dispersion. It is used in business to study the disparities of the distribution of wages, sales, production etc. In Economics it is useful to measure inequalities in the distribution of income. It is a graph down to a frequency distribution. While drawing the graph, cumulative percentage values of frequencies on X axis and cumulative percentage values of the variable on Y axis.

**Index Numbers:** Index numbers is a statistical device for measuring the changes in group of related variables over a period of time.

**Uses or Importance of index numbers.**

1. Index numbers measure trend values.
2. Index numbers facilitate for policy decisions.
3. Index numbers help in comparing the standard of living.
4. It measures changes in price level.
5. Index numbers are economic barometers.

**Limitations**

1. Index numbers are only approximate indicator.
2. All index numbers are not good for all purposes.
3. Index numbers are liable to be unissued.
4. Index numbers are specialized average and limitations of average also applicable to index numbers.

**Problems or Difficulties in the construction of index numbers**

1. Purpose of the index.
2. Selection of the lease period.
3. Selection of items.
4. Selection of an average
5. Selection of weights
6. Selection of appropriate source of data
7. Selection of suitable formula.

**Methods of constructing index numbers**

1)Unweighted or2)Weighted

**Un weighted or Simple index numbers**

Simple index numbers are those index numbers in which all items are treated as equally. Simple aggregate and simple average price relatives are the un weighted index numbers.

**(1)Simple Aggregate method**

$P_{01} = \frac{\sum P1}{\sum P0} \; x \; 100$

$P_{01}$ = index number

$P_1$ = Price for the current year

$P_0$ = Price for the base year.

**(2)Simple Average Price Relative Method**

Price index = $\frac{\sum I}{n}$

$I = \frac{P1}{P0}$ x 100, each item can be calculated.

**Weighted index numbers**

In this method quantity consumed is also taken into account. Such index are

1. Weighted aggregate method
2. Weighted Average of price relatives

**Weighted aggregate method**

This method is based on the weight of the prices of the selected commodities. Following are the commonly used methods:

1. Laspeyre's Method
2. Paasche's Method
3. Bowley-Dorbish Method
4. Fishers ideal method
5. Kelly's Methods

**Laspeyre's Method**

$P_{01} = \frac{\sum p1q0}{\sum p0q0} \; x \; 100$

$p_1$ = Price of the current year

$q_0$ = Quantity of the base year

$p_0$ = Price of the base year

**Paasche's Method**

$P_{01} = \frac{\sum p1q1}{\sum p0q1} \; x \; 100$

$q_1$ = Quantity of the current year

**Fishers Ideal Method**

$P_{01} = \sqrt{L \; x \; P \; x \; 100}$

L = Laspeyres method

P = Paasche's Method

$P_{01} = \sqrt{\frac{\sum p1q0}{\sum p0q0} \; x \; \frac{\sum p1q1}{\sum p0q1}} \; x \; 100$

**Bowley Doribish Method**

$P_{01} = \frac{L+P}{2}$

**Kelly's Method**

$P_{01} = \frac{\sum p1q}{\sum p0q} \; x \; 100$

$q = \frac{q0+q1}{2}$

**Weighted Average Price Relative Method**

Index number = $\frac{\sum IV}{\sum V}$

V = Weight

$I = \frac{p1}{p0}$ 100

**Consumer Price index number of cost of Living index number or Retail Price index number**

Consumer Price index number is also known as copy of Living Index number. It is the ration of the monetary expenditures of an individual which secure him the standard of living or total utility in two situations differing only in respect of prices.

**Methods of Constructing Consumer Price Index Number**

**Aggregate Expenditure Method**

Cost of living Index number = = $\frac{\sum p1q0}{\sum p0q0} \; x \; 100$

(2) **Family Budget Method**

Cost of Living Index = $\frac{\sum IV}{\sum V}$

**Time Series Analysis**

Time series is the arrangement of data according to the time of occurrence. It helps to find out the variations to the value of data due to changes in time.

**Components of Time Series**

**1)Secular Trend:** Trend may be defined as the changes over a long period of time. The significance of trend is greater when the period of time is very longer. important method of measuring trend are

1. Graphic Method
2. Semi Average Method
3. Moving Average Method
4. Method of Least Squares

2) **Seasonal Variations**:- Seasonal Variations are measured for one calendar year. It is the variations which occur some degree of regularity.

3) **Cyclical Variations**:- Cyclical variations are those variation which occur on account of business cycle. They are Prosperity, Decline, Depression and Recovery.

4) **Irregular fluctuations**:- One changes of variable could not be predicted due to irregular movements. Irregular movements are like changes in technology, war, famines, flood etc.

**Methods of Measuring Trend**

**(1)Graphic method :** This is the simplest method of measuring trend. Under this method original data are plotted on the graph paper. The plotted points should be joined, we get a curve. A straight line should be drawn through the middle area of the curve. Such line will describe tendency of the data.

**(2)Semi Average Method:-** The whole data are divided in to two parts and average of these are to be calculated. The two averages are to be plotted in the graph. The two points plotted should be joined so as to get a straight line. This line is called the ward live.

**(3)Method of Moving average:-** Under this method a series of successive average should be calculated from a series of values moving average may be calculated for 3,4,5,6 or 7 years periods.

4) **Method of Least Squares:** This is a popular method of obtaining trend line. The trend line obtained through this method is called line of best fit. One trend line is represented as $y = a + bx$

The value of **a** and **b** can be ascertained by solving the following two normal equations. $\Sigma y = Na + b\Sigma x$

$$\Sigma xy = a\Sigma x + b\Sigma X^2$$

Where **x** represents the time, **y** represents the value, **a** and **b** are constant and **N** represent total number.

When the middle year is taken as the origin, then $\Sigma x = 0$, then normal equation would be

$\Sigma xy = Na$

$\Sigma xy = b\Sigma x^2$

Hence a = $\frac{\sum xy}{\sum x^2}$

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**Study well...**